

QISS Kick-off, Hong-Kong, 16.1.2020

What is discrete covariance?

Pablo Arrighi

Aix*Marseille Univ.



* Recap

- * Discrete settings
- * Discrete covariance

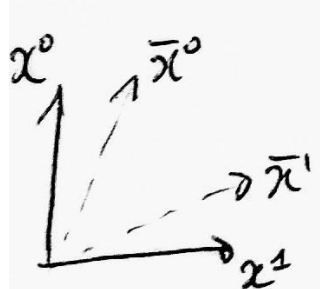


Recap > General covariance a.k.a diffeomorphism covariance

= *names of events*
coord. systems } *do not matter to* { *what happens*
physics laws

Phys. laws expressed/transforming as tensors...

...keep the same form. (transform. absorbed into phys. fields)



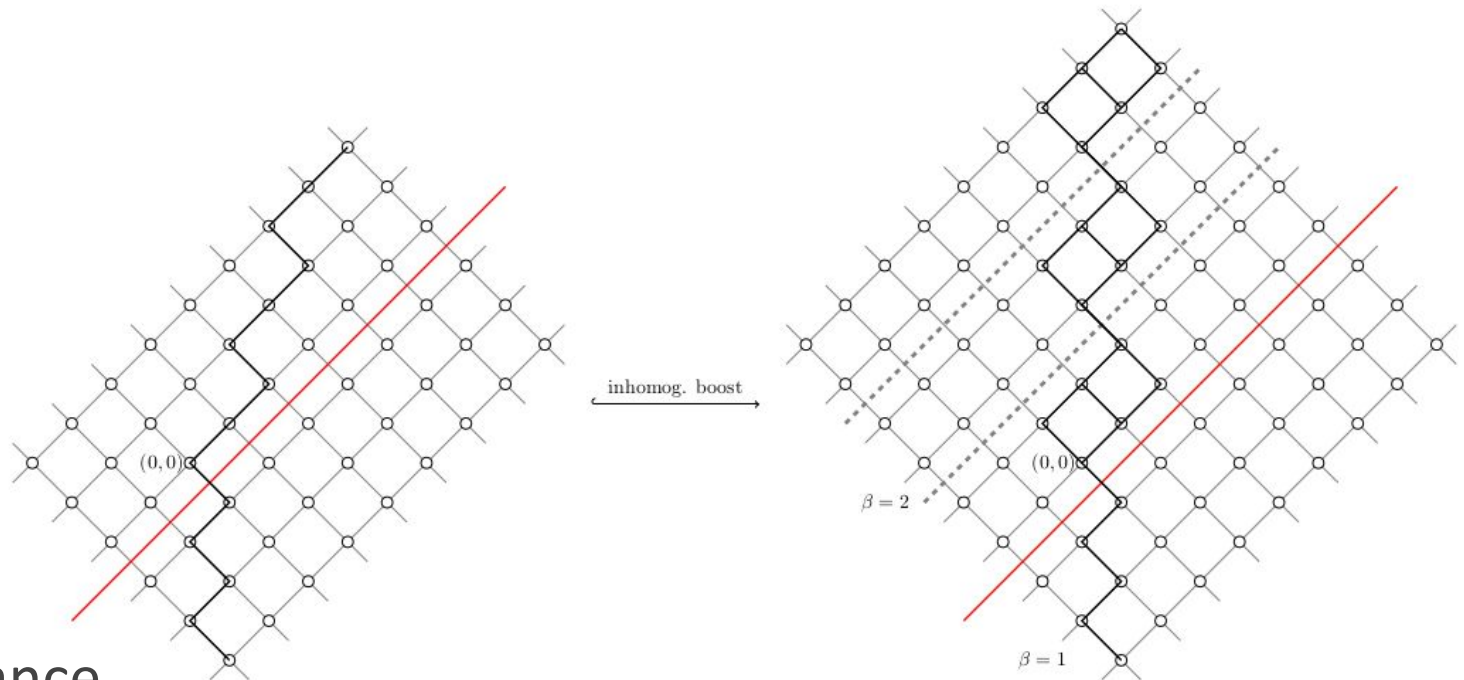
$$\begin{aligned}
 x^{\mu'} &\mapsto A^{\mu'}_{\mu} x^{\mu} = \bar{x}^{\mu'} & \sigma_{\nu} &\mapsto \sigma_{\nu'} (A^{-1})^{\nu}_{\nu'} = \bar{\sigma}_{\nu'} \\
 &\text{contravariant} & &\text{covariant} \\
 A^{\mu'}_{\mu} &= \frac{\partial \bar{x}^{\mu'}}{\partial x^{\mu}} & (A^{-1})^{\nu}_{\nu'} &= \frac{\partial x^{\nu}}{\partial \bar{x}^{\nu'}} & T^{\mu\dots}_{\nu\dots} &\mapsto A^{\mu'}_{\mu} \dots T^{\mu\dots}_{\nu\dots} (A^{-1})^{\nu}_{\nu'} \dots = \bar{T}^{\mu'}_{\nu'} \\
 \sigma \circ x &= \sigma_{\mu} x^{\mu} = \sigma A^{-1} A x = \bar{\sigma} \circ \bar{x}
 \end{aligned}$$

Simplest when based on scalars as action.

Recap > General principle of relativity

= Phys. laws same in every referential frame, incl. non-uniform.

Could have been implemented with strict subclass of diffs:

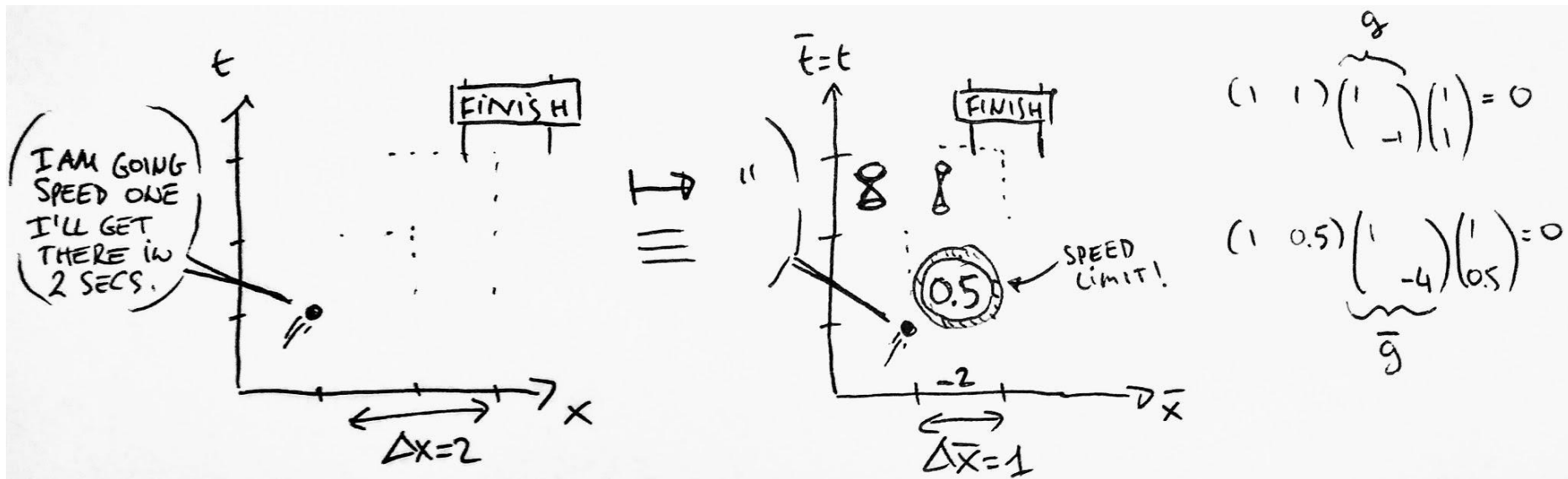


So, covariance

~ *Only phys. events and their interrelations matter.*

Recap > Covariance $\Rightarrow$ (possibly flat) metric

Any thy can be made covariant by adding a metric field.



Covariance $\nRightarrow$ Curved space

Recap > **Background independence $\Rightarrow$ Curved spacetime**

= Physics laws do not assume any fixed spacetime geometry, instead, they predict it.

How?

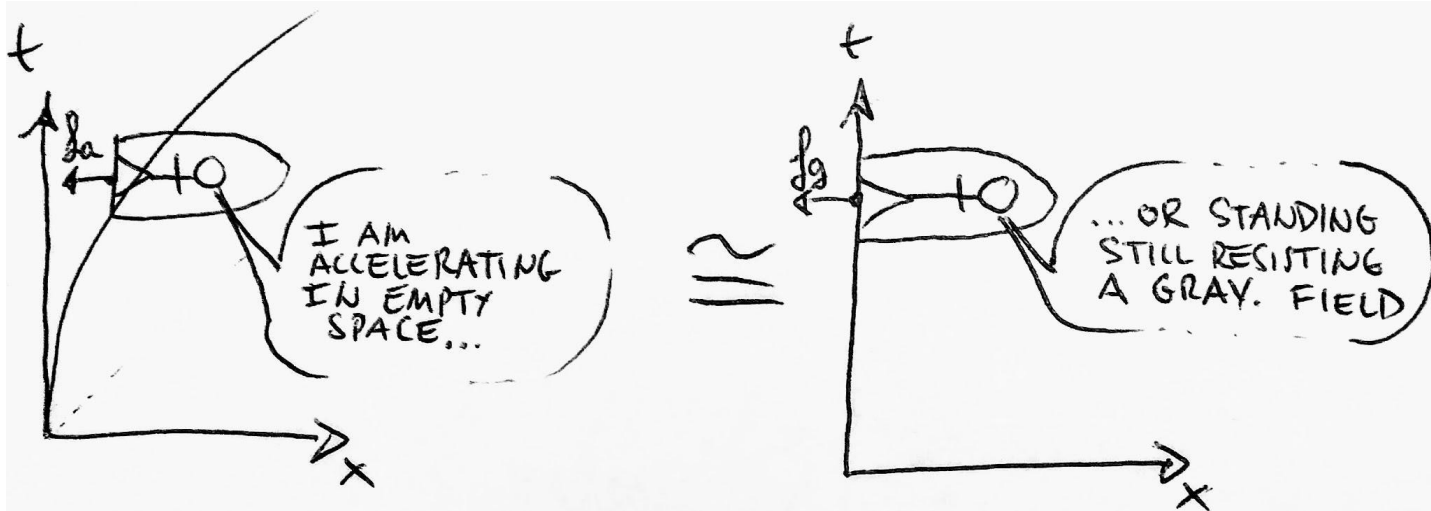
Background indep. does not prescribe evolution of the metric.
Need covariance back on the pitch.

Recap > Principle of equivalence $\Rightarrow$ Curved spacetime

Lab in shuttle accelerating to the right.

$\cong$ Lab still, resisting a grav. field from its left.

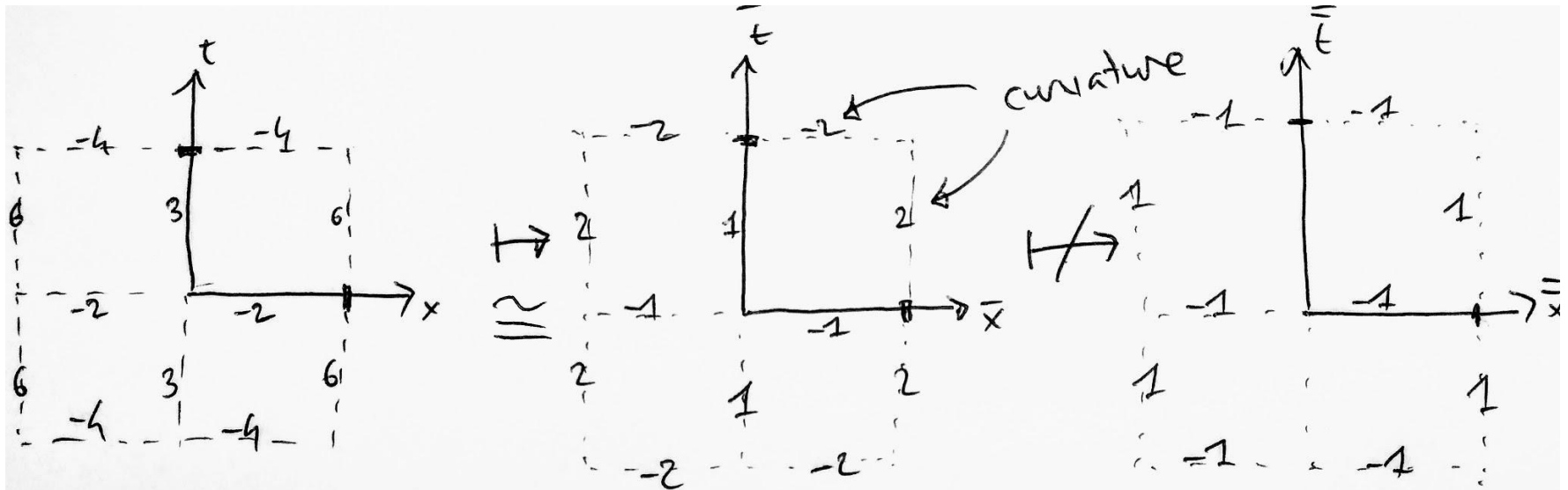
$\cong$ Lab still, within a modified metric field.



Grav. field carried by the metric. + grav. field sourced by the mass $\rightarrow$ mass curves space.

Recap > Covariance $\Rightarrow$ Metric ruled by curvature

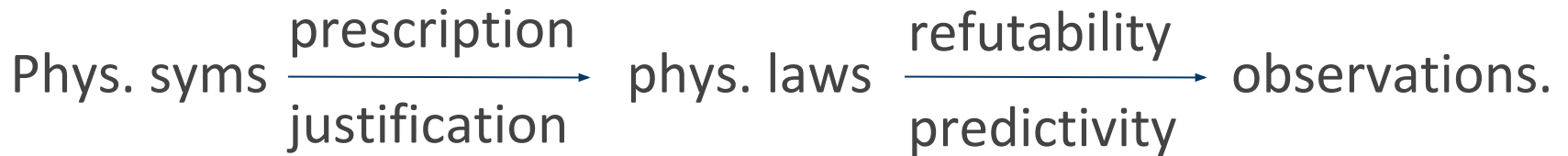
Diffs to geodesics frame, normalizes neighbourhood of a point.



The simplest phys. quantity resisting diffs is curvature R .

- $R = 0$ (flatness)
- R as action (Hilbert-Einstein action, EFEs)

Recap > Role of symmetries



- Covariance $\nRightarrow$ Curved space
- Background indep., Principle of eq. $\Rightarrow$ Curved space
- Covariance $\Rightarrow$ Metric, ruled by curvature

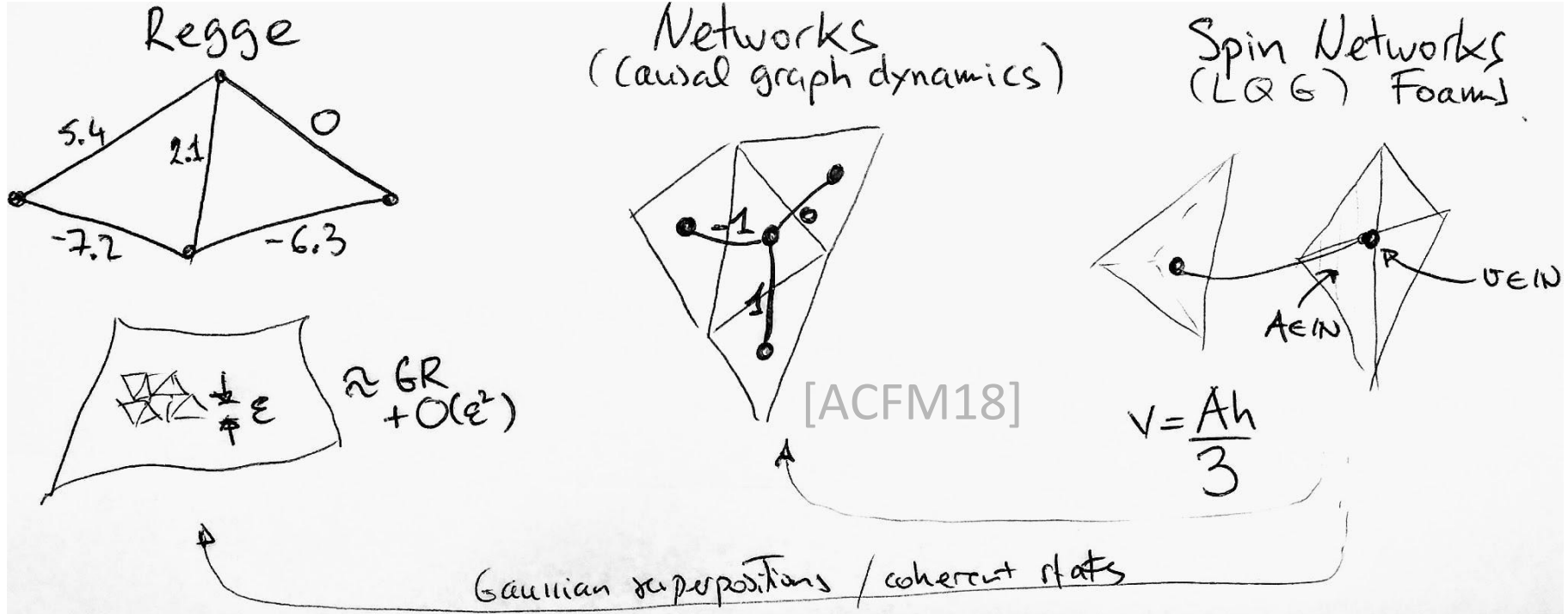
Covariance prescribes evolution of the metric.

Quantum gravity lacks such a direct prescription ?



- * Recap
- * **Discrete settings**
- * Discrete covariance

Discrete settings



Fuzziness



- * Recap
- * Discrete settings
- * **Discrete covariance**

Discrete covariance > **Vacuous !**

Discrete formalism capture just phys. events and interrelations.

Already modulo covariance. (This is why it's fuzzy ?)

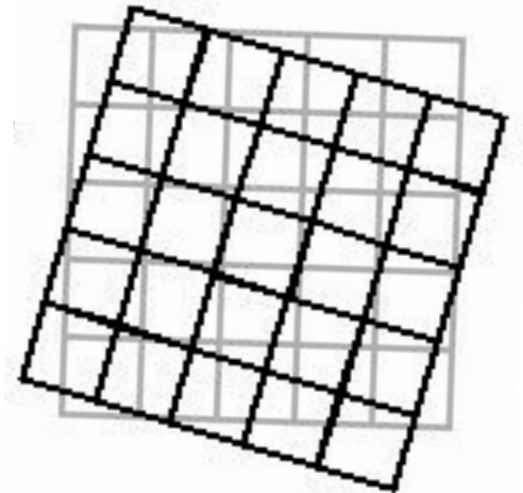
No other degree of freedom. (But naming issues...)

$\neq$ triangulations $\equiv \neq$ phys. situations.

Ex. $\neq$ triangulations of flat space $\cong$ moving a crystal.

(V0) Discrete covariance is vacuous/built-in.

(Background indep. is the important thing.)



Discrete covariance > In the continuum/classical limit

→ No prescription for evolution of the metric.

Trial & error, hoping "curvature as action emerges".

(V1) Covariance in some continuum/classical limit.

Not Einstein's path. Can we replay his method?

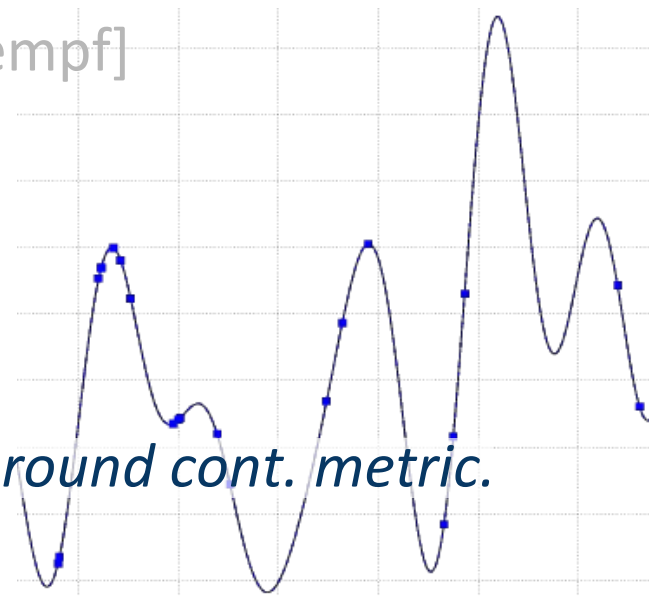
Natively discrete diffs → discrete covariance → Geodesics frame → Simplest phys. quantity.

Discrete covariance > **Sampling theorem**

"Spacetime could be simultaneously continuous and discrete in the same way that information can" [Kempf]

BW limited function $\equiv$ Finite samples

(V2) Discrete diffs are resampling background cont. metric.

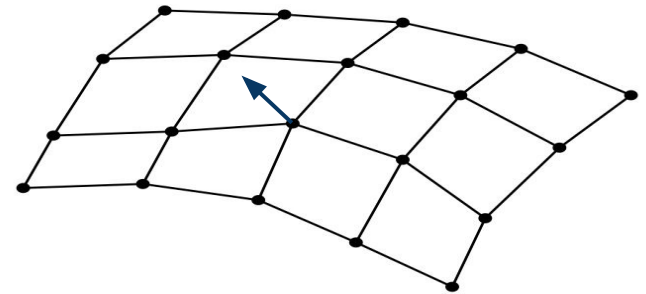


Adding a sample requires reconstructing the entire function.

→ No local gen. of discrete diffs.

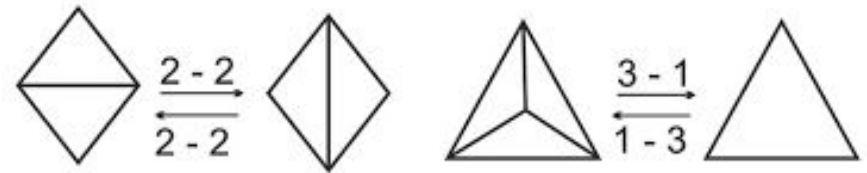
(Unless enough samples to do so locally... weird.)

Discrete covariance > **Local moves**



Gen. of diffs = just moving a point.

Gen. of discrete diffs $\stackrel{?}{=}$ Local, equiv. changes of triangulations.



Homeomorphic, so Pachner moves $\rightarrow$ **3-1 flatten spacetime.**

For Regge, vertex displacements $\rightarrow$ **$O(\epsilon^2)$ only, except when flat.**



Fuzzyness, polynomial interpolations, could save the day?

(V3) Diffs are length-preserving, curvature-preserving local moves, using poly interpolations so error kept within $O(\epsilon^2)$.

Discrete covariance > Refinement

3-1 moves destroy information, coarse-graining.

→ restrict to 1-3 / stellar subdivisions, fine-graining.

~ (skewed) scale invariance, Ex. Lorentz-covariance : [AFF14]

Hit ℓ_p , are further refinements allowed ?

No. Left with finite set of phys. neighbourhoods.

Yes. with $d=0$ (LQG). Not sure this helps.

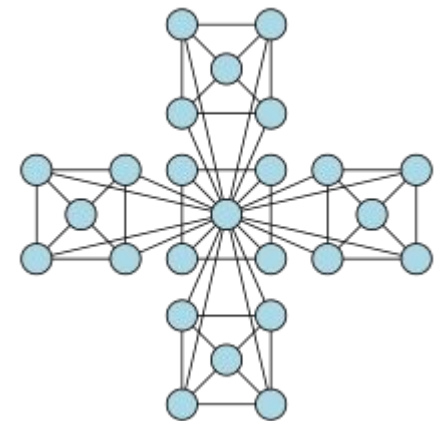
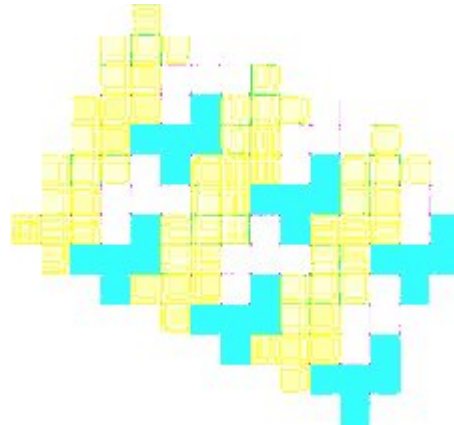
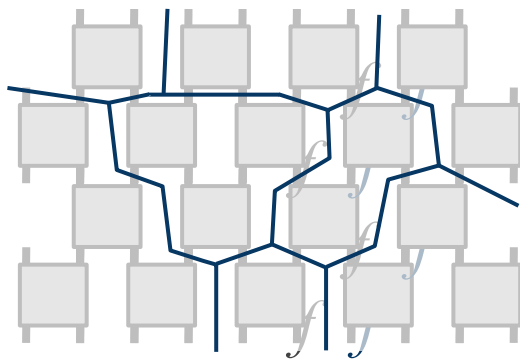
Yes. Compatible with bounded density

~ Further sampling BW-limited function

Discrete covariance > **Refinement**

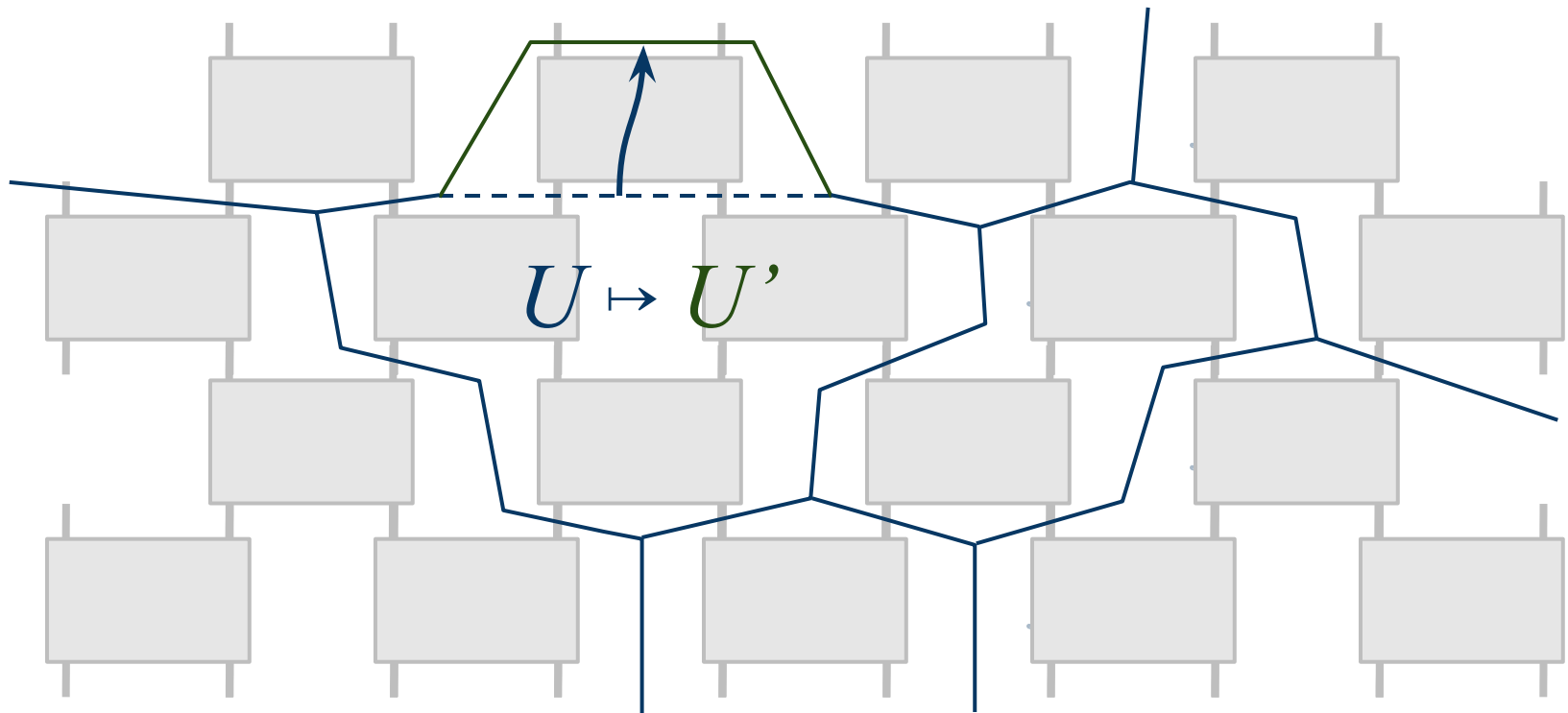
(V4) Discrete diffs are length-preserving, curvature-preserving, refinements, using poly interpolations so error kept within $O(\varepsilon^2)$.

Ex. Lightlike grid of QCA cells polyomino
Timelike grid of MPU tensor network cells $\mapsto$
 Networks nodes subnetworks



Discrete covariance > Tensorial representation

Refine, move boundaries, coarse-grain.

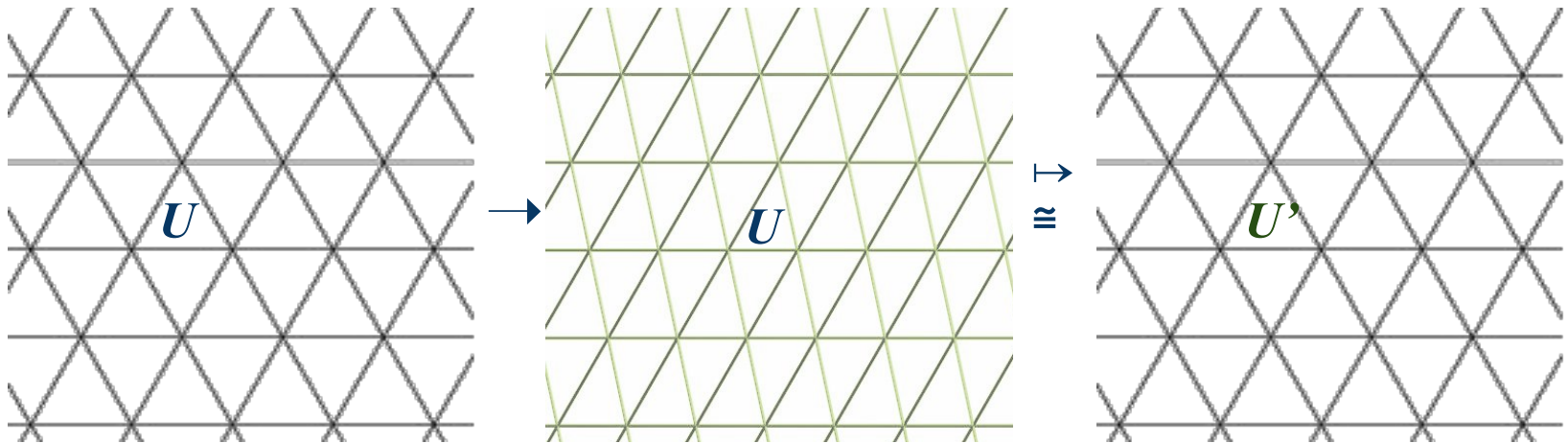


(V5) Discrete diffs represented as acting on the boundaries of tensors, standing for atomic spacetime regions. [Oeckl]

Discrete covariance > Tensorial representation

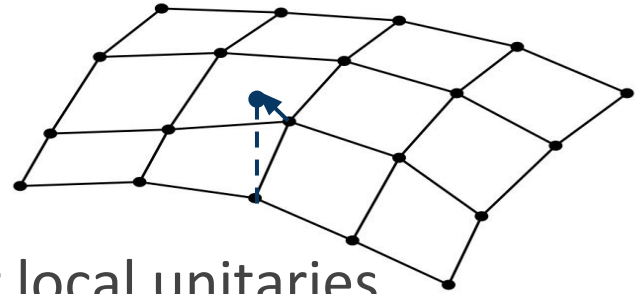
Ex. Duality. [ADMP19]

Skewed geom. Dirac QCA $\cong$ Straight geom. modified Dirac QCA



→ Smooth skewing, o/wise spurious term in Curved Dirac Eq.

Discrete covariance > Gauge



GR $\notin$ Yang-Mills gauge theory = inv. under local unitaries

GR gen. of diffs act

- on neighbourhoods $\rightarrow$ Use half-distances ?
- non-unitarily $\rightarrow$ Moving boundary over refinement ?

Dynamically, evolution combined with loc. specified diff. takes:

$$(t,x) \text{ with (Lapse, Shift)} \mapsto (t+1+\text{Lapse}, x+\text{Shift})$$

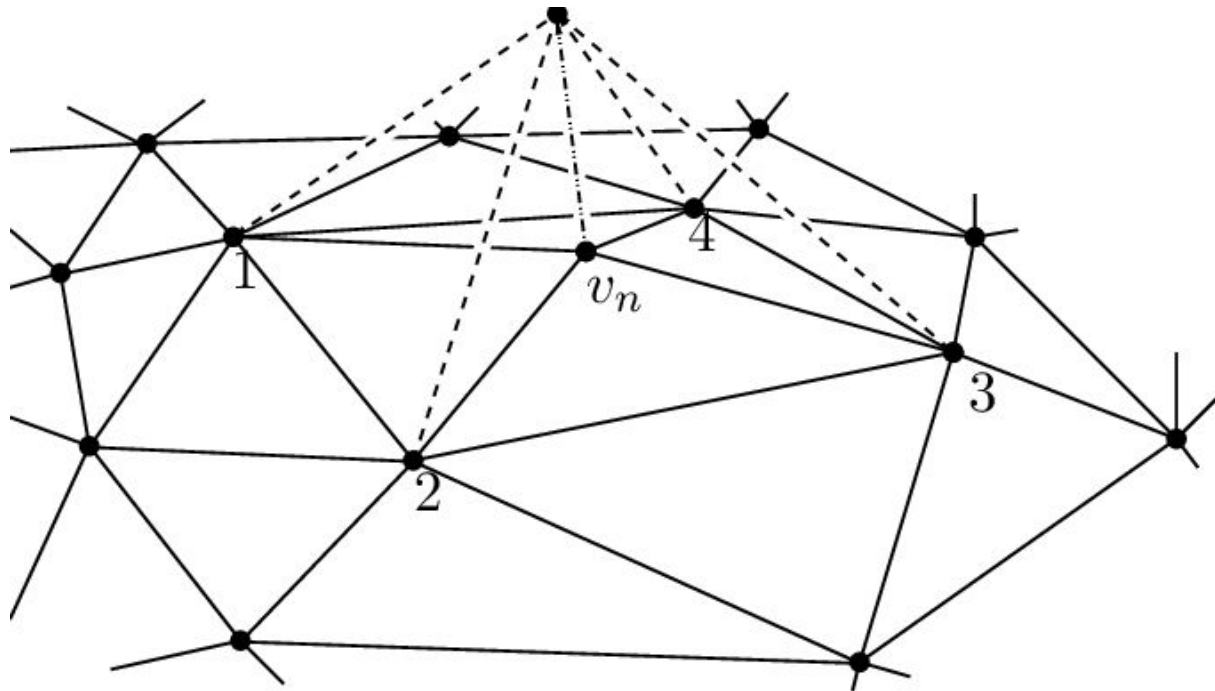
(V6) Discrete covariance is a gauge theory with respect to lapse-and-shift gauge transformation.

Discrete covariance > **Gauge**

Ex. Regge curvature as action fully determines metric

→ Drop a few eqs to get Lapse-Shift degrees of freedom

[Sorkin]



Conclusion

Need discrete covariance to prescribe evolution, via :

Discrete diffs $\rightarrow$ discrete covariance $\rightarrow$ Geodesics frame $\rightarrow$
Simplest phys. quantity.

Moving boundaries over refinement suggest
acting on boundaries of tensorial chunks of spacetime
in a gauge theoretical way.

Quantum reference frames ?

